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Approximate Controllability of Fractional Delay Dynamic Inclusions with Nonlocal Control Conditions

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Abstract. We introduce a nonlocal control condition and the notion of approximate controllability for fractional order quasilinear control inclusions. Approximate controllability of a fractional control nonlocal delay quasilinear functional differential inclusion in a Hilbert space is studied. The results are obtained by using the fractional power of operators, multivalued analysis, and Sadovskii's fixed point theorem. Main result gives an appropriate set of sufficient conditions for the considered system to be approximately controllable. As an example, a fractional partial nonlocal control functional differential inclusion is considered.

Keywords. approximate controllability; control theory; multivalued maps; fractional dynamic inclusions; fractional power; fixed points; semigroup theory.

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1 Introduction

We are concerned with the fractional delay quasilinear control inclusion

$$D_t^{\alpha}[u(t) - g(t, u(\sigma(t)))] \in Au(t) + \int_0^t f(t, s, B_1\mu_1(\delta(s)))ds \tag{1}$$

subject to the nonlocal control condition

$$u(0) + h(u(t)) = B_2\mu_2(t) + u_0, \tag{2}$$

where the unknown $u(\cdot)$ takes its values in a Hilbert space H with norm $\|\cdot\|$, D_t^{α} is the Caputo fractional derivative with $0 < \alpha \leq 1$ and $t \in J = [0, a]$. Let A be a closed linear operator defined on a dense domain $D(A)$ in H into H that generates an analytic semigroup $Q(t)$, $t \geq 0$, of bounded linear operators on H and $u_0 \in D(A)$. We assume that $\{B_i : U \to H,\ i = 1, 2\}$ is a family of bounded linear operators, the control functions μ_i, $i = 1, 2$, belong to the space $L^2(J, U)$, a Hilbert space of admissible control functions with U as Hilbert space, and $\sigma, \delta : J \to J'$ are delay arguments, $J' = [0, t]$. It is also assumed that $g : J \times H \to H$ and $h : C(J' : H) \to H$ are given abstract functions and $f : \Delta \times H \to H$ is a multi-valued map, $\Delta = \{(t, s) : 0 \leq s \leq t \leq a\}$.

2 Main results

We obtain existence and approximate controllability results for the fractional nonlocal control inclusion (1)–(2). We consider the following hypotheses:

(H$_1$) There exists a constant $p \in (0,1)$ such that the function $g(\cdot,\cdot)$ maps $[0,a] \times H_q$ into H_{p+q} and $A^p g : [0,a] \times H_q \to H_q$ satisfies a Lipschitz condition, that is, there exists a constant $L_1 > 0$ such that

$$\|A^p g(t_1,u_1) - A^p g(t_2,u_2)\|_q \leq L_1 \left(|t_1 - t_2| + \|u_1 - u_2\|_q\right) \tag{3}$$

for any $0 \leq t_1, t_2 \leq a$, $u_1, u_2 \in H_q$.

(H$_2$) For any $u(\cdot) \in \Omega, u_a \in H$, we take the controls

$$\mu_1 = B_1^* T_\alpha^*(a-t)\mathcal{R}(\lambda,\Gamma_{0,1}^a)P(u(\cdot)), \quad \mu_2 = B_2^* S_\alpha^*(a)\mathcal{R}(\lambda,\Gamma_{0,2}^a)P(u(\cdot)). \tag{4}$$

Theorem 1 *If (H$_1$)–(H$_2$) are satisfied and $\lambda\mathcal{R}(\lambda,\Gamma_{0,i}^a) \to 0$ in the strong operator topology as $\lambda \to 0^+$, $i = 1,2$, then the nonlocal-control fractional delay system (1)–(2) is approximately controllable on J.*

Proof. We use the existence result of mild solutions, it is easy to see that F^λ has a fixed point in Ω_r for any $\lambda \in (0,1)$. This implies that there exists $\overline{u}^\lambda \in F^\lambda(\overline{u}^\lambda)$, that is, there is $\overline{v}^\lambda \in S_{f,\overline{\mu_1}^\lambda}$ such that

$$\begin{aligned}\overline{u}^\lambda(t) = {} & S_\alpha(t)\left[B_2\overline{\mu_2}^\lambda(t) + u_0 - h\left(\overline{u}^\lambda\right) - g\left(0,\overline{u}^\lambda(\sigma(0))\right)\right] + g\left(t,\overline{u}^\lambda(\sigma(t))\right) \\ & + \int_0^t (t-s)^{\alpha-1}\left\{AT_\alpha(t-s)g\left(s,\overline{u}^\lambda(\sigma(s))\right) + T_\alpha(t-s)\left[\overline{v}^\lambda(\delta(s)) + B_1\overline{\mu_1}^\lambda(s)\right]\right\}ds.\end{aligned}$$

Moreover, by the assumption that we have $\lambda\mathcal{R}(\lambda,\Gamma_{0,i}^a) \to 0$ in the strong operator topology as $\lambda \to 0^+$, $i = 1,2$, we ensure that $\|u_a - \overline{u}^\lambda(a)\| \to 0$ as $\lambda \to 0^+$. Therefore, the fractional dynamic inclusion (1)–(2) is approximately controllable on J. ■

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